



Oxford Cambridge and RSA

**Monday 05 October 2020 – Afternoon**

**AS Level Further Mathematics A**

**Y531/01 Pure Core**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- This document has **4** pages.

**ADVICE**

- Read each question carefully before you start your answer.

Answer **all** the questions.

**1 In this question you must show detailed reasoning.**

Use an algebraic method to find the square roots of  $-77 - 36i$ . [6]

**2 P, Q and T are three transformations in 2-D.**

P is a reflection in the  $x$ -axis. **A** is the matrix that represents P.

(a) Write down the matrix **A**. [1]

Q is a shear in which the  $y$ -axis is invariant and the point  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  is transformed to the point  $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ . **B** is the matrix that represents Q.

(b) Find the matrix **B**. [2]

T is P followed by Q. **C** is the matrix that represents T.

(c) Determine the matrix **C**. [2]

$L$  is the line whose equation is  $y = x$ .

(d) Explain whether or not  $L$  is a line of invariant points under  $T$ . [2]

An object parallelogram,  $M$ , is transformed under T to an image parallelogram,  $N$ .

(e) Explain what the value of the determinant of **C** means about

- the area of  $N$  compared to the area of  $M$ ,
- the orientation of  $N$  compared to the orientation of  $M$ .

[3]

**3 In this question you must show detailed reasoning.**

The complex number  $7 - 4i$  is denoted by  $z$ .

(a) Giving your answers in the form  $a + bi$ , where  $a$  and  $b$  are rational numbers, find the following.

(i)  $3z - 4z^*$  [2]

(ii)  $(z + 1 - 3i)^2$  [2]

(iii)  $\frac{z+1}{z-1}$  [2]

(b) Express  $z$  in modulus-argument form giving the modulus exactly and the argument correct to 3 significant figures. [3]

(c) The complex number  $\omega$  is such that  $z\omega = \sqrt{585}(\cos(0.5) + i \sin(0.5))$ .

Find the following.

- $|\omega|$
- $\arg(\omega)$ , giving your answer correct to 3 significant figures [3]

**4 You are given the system of equations**

$$\begin{aligned} a^2x - 2y &= 1 \\ x + b^2y &= 3 \end{aligned}$$

where  $a$  and  $b$  are real numbers.

(a) Use a matrix method to find  $x$  and  $y$  in terms of  $a$  and  $b$ . [4]

(b) Explain why the method used in part (a) works for all values of  $a$  and  $b$ . [2]

**5 In this question you must show detailed reasoning.**

The cubic equation  $5x^3 + 3x^2 - 4x + 7 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .

Find a cubic equation with integer coefficients whose roots are  $\alpha + \beta$ ,  $\beta + \gamma$  and  $\gamma + \alpha$ . [7]

6 Prove that  $n! > 2^{2n}$  for all integers  $n \geq 9$ . [5]

7 The equations of two **intersecting** lines are

$$\mathbf{r} = \begin{pmatrix} -12 \\ a \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \quad \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$$

where  $a$  is a constant.

(a) Find a vector,  $\mathbf{b}$ , which is perpendicular to both lines. [2]

(b) Show that  $\mathbf{b} \cdot \begin{pmatrix} -12 \\ a \\ -1 \end{pmatrix} = \mathbf{b} \cdot \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix}$ . [2]

(c) Hence, or otherwise, find the value of  $a$ . [2]

8 Two loci,  $C_1$  and  $C_2$ , are defined by

$$C_1 = \left\{ z : |z| = |z - 4d^2 - 36| \right\}$$

$$C_2 = \left\{ z : \arg(z - 12d - 3i) = \frac{1}{4}\pi \right\}$$

where  $d$  is a real number.

(a) Find, in terms of  $d$ , the complex number which is represented on an Argand diagram by the point of intersection of  $C_1$  and  $C_2$ .

[You may assume that  $C_1 \cap C_2 \neq \emptyset$ .] [6]

(b) Explain why the solution found in part (a) is not valid when  $d = 3$ . [2]

**END OF QUESTION PAPER**

**OCR**

Oxford Cambridge and RSA

**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.